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LITTORAL COMBAT SHIP CREW SCHEDULING

by

Van R. Fitzsimmons

March 2015

Thesis Advisor:	Javier Salmeron
Second Reader:	Nedialko Dimitrov

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The Littoral Combat Ship (LCS) is a naval combatant designed to operate in the littoral regions. Twenty-four LCSs will be built over the next five years employing a crew rotation concept where three crews rotate between two ships. During the construction period, an experienced crew must be assigned, which disrupts the desired crew rotation in ships already built.

This thesis develops "LCS Scheduler" (LCSS), a mathematical optimization model using a mixed-integer, linear program (MIP) to aid in assigning LCS crews to LCS ships. LCSS's objective is to minimize the penalty associated with assigning crews outside of their desired ship pairing and/or extending them beyond four months in a phase.

Results are compared based on solve time and penalty value. The MIP solution has the best quality. Yet, even for a shorter-than-desired time horizon, it takes many hours of computation. Rolling horizon is a heuristic approach that produces a full, long-term schedule in under an hour but requires manual modifications to misaligned crews. Fix-and-relax is a more-elaborate heuristic with potential benefits to crew alignment for longer-range schedules. The planner must balance solve time and solution quality when determining the approach to LCSS.

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LITTORAL COMBAT SHIP CREW SCHEDULING

Van R. Fitzsimmons
Lieutenant Commander, United States Navy
B.S., Cornell University, 1999

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Author: Van R. Fitzsimmons

Approved by: Javier Salmeron
Thesis Advisor

Nedialko Dimitrov
Second Reader

Robert F. Dell
Chair, Department of Operations Research

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TABLE OF CONTENTS

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LIST OF FIGURES

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LIST OF TABLES

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LIST OF ACRONYMS AND ABBREVIATIONS

CA	Crew Assignment
CMA	Crew Move Aboard
CONUS	Continental United States
CPLEX	An optimization software package
DON	Department of the Navy
F&R	Fix and Relax
GAMS	General Algebraic Modeling System
LCS	Littoral Combat Ship
LCSRON	Littoral Combat Ship Squadron (or its Commander)
LCSS	Littoral Combat Ship Scheduler (an optimization model)
ln	Natural logarithm function
MH-60R	A twin-engine, medium lift, utility or assault helicopter
MIP	Mixed-Integer, Linear Program
PCU	Pre-Commissioning Unit
RH	Rolling Horizon
USA	United States of America
USN	United States Navy
USS	United States Ship

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EXECUTIVE SUMMARY

The Littoral Combat Ship (LCS) is a small, maneuverable and reconfigurable naval vessel designed to operate in the littoral regions of the world. To reduce the number of ships required to satisfy Department of Defense deployed presence requirements for LCS, a crew rotation concept is employed where three LCS crews are assigned to two LCS on a four-month rotation plan. This rotational plan is known as 3-2-1. New LCSs will be built over the next five years. During the construction process, the LCS Commander (LCSRON) requires that a crew that has completed a full deployment cycle is assigned to a new ship to conduct initial sea trials, major inspections and transit to homeport. This will disrupt the crew rotation flow, but LCSRON desires a method to minimize these disruptions while satisfying the experience requirement.

This thesis develops “LCS Scheduler” (LCSS), a mathematical optimization model using a mixed-integer, linear program (MIP) to aid LCSRON’s assignment of LCS crews to LCS ships. LCSS’s objective is to minimize the penalty associated with assigning a crew to a ship outside of its desired ship pairing or extending a crew beyond four months in a phase. In addition, LCSS satisfies the required training flow prior to operational deployments and experience requirements of new ship construction. LCSS is designed to guide the scheduler by producing an initial quality solution that can be modified to account for other intangible requirements of the command.

The LCSS MIP for a three-year time horizon is computationally intractable. Since the long-range schedule for deployable ships is uncertain, it is reasonable to place more emphasis on short-term obligations. Two optimization-based heuristic approaches, rolling horizon (RH) and fix-and-relax (F&R), are used to reduce solve times.

RH partitions LCSS into sub-problems of significantly smaller length than the original MIP. Each sub-problem in RH takes a myopic view of the overall schedule. This leads to a significant reduction in solve time, but does not allow consideration of out-year schedules.

F&R also solves a number of sub-problems, but considers all time periods during each sub-problem. Variables are divided into integer, relaxed (i.e., continuous), or “fixed data,” depending on the sub-problem. Solve times for F&R are longer than for RH because of the relaxation afforded to out-year schedules.

RH and F&R employ an “unfixed overlap period” between successive sub-problems to mitigate end-effects. This allows the incumbent sub-problem to change some of the past crew-ship assignments to better schedule incumbent requirements.

Results are compared based on solve time and penalty value. LCSS MIP solve times increase exponentially making longer range schedules require too much computational time to be useful for the scheduler. A desirable goal would be to generate a solution for up to three years, but the computational time for the MIP formulation is unacceptable. Thus, we generate a MIP schedule for up to 30 months. In the case of RH and F&R there is no horizon limit as long as we keep our partitions short. Specifically, we generate a 40-month schedule by using a three-sub-problem partition.

For obvious reasons, the 30-month MIP solution has the best quality when compared to the 30 first months of a 40-month RH or F&R solutions. These produce 60% and 80% more incidents of misaligned crews and/or extensions, respectively. The MIP solution takes considerably longer time to be produced (approximately 6.5 hours) than the RH solution (approximately 1.25 hours), but that can be acceptable depending on the scheduler needs. F&R produces superior long-term schedules when compared to a similar-length RH schedule, but it also takes longer time to solve (7 hours).

The operational planner must balance the requirement of the desired schedule with the implementation approach of LCSS. RH and F&R offer the planner a full, long-term schedule requiring some manual modifications, while MIP is not able to provide a solution in a reasonable amount of time for a planner. However, if long-term implications of a schedule are not as important as short-term optimal assignment then a shorter MIP can be solved that provides a guaranteed optimal solution that requires minimal manual modifications.

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I. INTRODUCTION

As of 2015, the Littoral Combat Ship (LCS) program has been operational for seven years with a small number of vessels. Over the next five years, the production of LCSs will increase the fleet from four to twenty-four ships. Accepting these ships from the shipyard and conducting the initial inspections requires experienced crews familiar with the operational requirements of LCS. The rotational crew concept of the LCS program requires a long-range schedule that can balance both acceptance and operational requirements. This thesis presents LCS Scheduler (LCSS), a mixed-integer, linear program (MIP) that optimizes the assignment of LCS crews to LCS ships. Specifically, LCSS minimizes penalties for assigning crews outside of their designated ship pairing or for extending them beyond their desired time in a phase, while ensuring all crew training prerequisites occur.

In January 2015, U.S. Secretary of the Navy Ray Maybus announced that the LCS would be re-designated as a Frigate with no changes to the ship design or the program of record. However, for purposes of this document, the original LCS designation will be used.

A. BACKGROUND

Twenty-first century naval warfare can no longer be approached with the sole mentality of large fleet-on-fleet engagements in the tradition of World War II or Cold War era navies. The ability to respond to global "hot spots" requires a Navy that is capable of effectively operating in contested littoral regions. To this end, in 2001 the Department of the Navy (DON) announced the LCS project. The LCS concept proposes a fleet of small, maneuverable ships that could be easily reconfigured to specialize in a variety of littoral combat missions. An LCS should be able to operate in contested waters against threats commonly found in those regions: enemy mines, submarines and swarm boats.

To accomplish this wide variety of littoral mission sets the LCS has multiple mission packages that can be rapidly installed on the ship and tailored to the specific primary mission. There are three mission packages for the LCS that support mine countermeasures, surface warfare and anti-submarine warfare. The mission packages are further subdivided into mission modules, mission crew detachments and aircraft. Mission modules provide systems and support equipment specifically tailored to the operational task. They are designed to fit into standard support containers to enable prepositioning around the globe for quick reaction to changing operational conditions. Mission crews are a small complement of sailors designed to augment the core LCS crew and operate the mission module. Also, LCS is capable of embarking an MH-60R helicopter detachment to augment any mission package.

The typical acquisition process begins by evaluating numerous industry selections to determine which one best satisfies the goals of the program. Usually one winning bid would be chosen for production. However, during this phase of the LCS program, it was determined that two contractors would produce two different LCS variants. The Freedom variant LCS (see Figure 1) is produced by Lockheed-Martin at its shipyard in Marinette, Wisconsin, and is identified by odd-numbered designations (LCS 1, LCS 3, etc.). The Freedom variant LCS is characterized by its single hull design. General Dynamics, Bath Iron Works and Austal USA were awarded the contract to produce LCS ships of the Independence variant (see Figure 2). This variant is built in Mobile, Alabama and is designated with even numbers (LCS 2, LCS 4, etc.). The Independence variant is readily identified by its characteristic trimaran hull.

These two ship variants neither have common design characteristics (see Table 1) nor systems and therefore crews trained for a specific variant cannot operate the other variant without re-training and certification.

Significant improvements in shipboard technology and automation have been made since the last class of Navy surface combatants was fielded. These improvements made it possible to reduce the size of the crew, and LCS is designed to function with a significantly smaller crew complement of 50 sailors when compared with current U.S. Navy (USN) surface combatants. The implementation of this reduced crew manning on *USS Freedom* resulted in an evaluation by the Chief of Naval Operations on workload and manning levels. It was determined that future LCS ships would be designed to accommodate a larger crew complement of 98 sailors. This expands the core crew to 53 plus the mission package detachment and the aviation detachment.

Figure 1. *USS Freedom* (LCS-1) (from http://en.wikipedia.org/wiki/USS_Freedom_(LCS-1))

Table 1. Comparison of the design characteristics of LCS variants and list of LCS ships (after http://www.navy.mil/navydata/fact_display.asp?cid=4200&tid=1650&ct=4)

	Freedom variant	Independence variant
Builder	Lockheed Martin	General Dynamics, Austal USA
Length	387.6 ft	418.6 ft
Beam	57.7 ft	103.7 ft
Displacement	3,400 MT	3,100 MT
Draft	14.1 ft	14.4 ft
Speed	40+ knots	40+ knots
Ships	USS Freedom (LCS 1) USS Forth Worth (LCS 3) PCU Milwaukee (LCS 5) PCU Detroit (LCS 7) PCU Little Rock (LCS 9) PCU Sioux City (LCS 11) PCU Wichita (LCS 13) PCU Billings (LCS 15)	USS Independence (LCS 2) USS Coronado (LCS 4) PCU Jackson (LCS 6) PCU Montgomery (LCS 8) PCU Gabrielle Giffords (LCS 10) PCU Omaha (LCS 12) PCU Manchester (LCS 14) PCU Tulsa (LCS 16)

Figure 2. *USS Independence* (LCS-2) (from http://en.wikipedia.org/wiki/USS_Independence_(LCS-2))

The program is also designed with a crew rotation concept in mind to allow for maximum deployment time while still maintaining periodic crew training and readiness. The USN implements a crew rotation concept on many of its ships. Most notably is the Blue-Gold system on ballistic missile submarines, but rotation is also used on coastal patrols ships and mine counter-measures ships. The results of this concept provide significantly more overseas presence for each ship as well as a reduction in the required number of ships to support operational requirements. The Congressional Budget Office (Labs, 2007) estimates that without a crew rotation concept the Navy will need to buy 30 additional LCS to meet the “forward presence” requirements. The specific cost savings are dependent on the type of rotational system employed. LCS’s implementation of this program is known as 3-2-1, where three crews are assigned to two ships with one ship always deployed. This requires fewer crews than a Blue-Gold structure and puts a higher importance on efficient scheduling of crews.

B. CREW ROTATION CONCEPT

During the construction process, a LCS passes numerous milestones laid out by the LCS Program Manager (2013) (see Figure 3). When a ship attains the “Builder’s Trial Start” milestone in the shipyard, a crew of sailors must be available, but these sailors do not need to be experienced on the platform.

		current as of:	9-Oct-13	
Hull	LCS 1	LCS 2	LCS 3 [4]	LCS 4 [4]
Yard	LM / MMC	GD / Austal USA	LM / MMC	GD / Austal USA
FY	2005	2006	2009	2009
Name	FREEDOM	INDEPENDENCE	FORT WORTH	CORONADO
CO / PCO blue/gold	CDRs Thiess / [illegible]	CDRs Gagliano / [illegible]	CDRs Capps / Kim	CDRs Kochendorfer / Johnston
Homeport	San Diego,CA	San Diego,CA	San Diego,CA	San Diego,CA
Fleet	PAC	PAC	PAC	PAC
UIC / PCU UIC	20126	20127	20130	20131
Award Date	15-Dec-04	14-Oct-05	23-Mar-09	1-May-09
Start Fab	5-Feb-05	14-Nov-05	21-Apr-09	3-Oct-09
Lay Keel	2-Jun-05	19-Jan-06	11-Jul-09	17-Dec-09
Launch	23-Sep-06	28-Apr-08	4-Dec-10	11-Jan-12
Christening	23-Sep-06	5-Oct-08	4-Dec-10	14-Jan-12
Builders' Trial Start	9-Jul-08	15-Jun-09	12-Sep-11 [5]	25-Mar-13
Builders' Trial End	3-Aug-08	18-Oct-09	21-Oct-11 [5]	27-Jul-13
Acceptance Trial Start	17-Aug-08	13-Nov-09	30-Apr-12	19-Aug-13
Acceptance Trial End	21-Aug-08	19-Nov-09	4-May-12	23-Aug-13
Delivery (plan)	18-Sep-08	18-Dec-09	6-Jun-12	27-Sep-13
Delivery (contract)			23-Aug-12	01-Mar-13
IPDA Start	13-Jan-09	19-Dec-09	N/A [3]	28-Sep-13
IPDA End	5-Apr-09	19-Mar-10	N/A	26-Nov-13
Crew Move Aboard	18-Sep-08	2-Feb-10	6-Jun-12	25-Nov-13
PDA Start	Sep-08	10-Jan-10	7-Jun-12	27-Nov-13
Complete Fitout	1-Oct-08	1-Feb-10	5-Aug-12	Jan-14
PDA End	Oct-08	10-Mar-10	5-Aug-12	Jan-14
Sailaway	24-Oct-08	26-Mar-10	5-Aug-12	Jan-14
Commissioning	8-Nov-08	16-Jan-10	22-Sep-12	Apr-14
Site	Milwaukee, WI	Mobile, AL	Galveston, TX	Coronado, CA
CSSQT Start	N/A	N/A	Oct-12	Mar-14
CSSQT End	N/A	N/A	Sep-13	Aug-14
FCT Start	22 May-12 [1]	TBD [4]	2-Apr-10	Jun-14
PSA Start	27-Jun-11 [2]	4-Sep-12 [3]	1-May-13	Jul-14
PSA Yard	BAE Systems SDSR [1]	NASSCO	Lockheed Martin	TBD
PSA End	24-Oct-11 [2]	5-Apr-13 [3]	26-Jul-13	Dec-14
SCN OWLD	N/A	N/A	31-Aug-13 [10]	Dec-14

Figure 3. Partial LCS production schedule with major milestones (from LCS Program Manager, 2013)

If this is the first ship in a new pair, the Navy funds two crews for the ship. One of these newly funded crews will be assigned to the ship in the Builder’s Trial phase. The other crew is available for tasking and assignment by the LCS Squadron (LCSRON). This second crew can begin the shore-based training process in preparation for an on-hull period with the ship once it completes its time in the shipyard. If this is the second ship in a new pair, only one crew is established.

The crew rotation concept starts with two LCS hulls being designated as a *pair*. For example, USS *Independence* (LCS 2) and USS *Coronado* (LCS 4) comprise one such pair. These two ships have three LCS crews (Crew 201, Crew 202, and Crew 203) assigned to them. The steady-state goal of the crew-ship rotation will have one ship forward-deployed for one year while the other ship remains in homeport in San Diego (see Figure 4). When a ship is in homeport it can undergo scheduled maintenance and upkeep. The forward deployed LCS is not at sea for the entire year. Currently, all LCSs forward deploy to Changi Naval Base in Singapore, home to Commander, Logistics Group Western Pacific to support U.S. Seventh Fleet. This base is capable of supporting visiting U.S. Navy ships which allows LCS to maintain an improved materiel condition while deployed, but does not offer the full depot-level support of San Diego.

<table>
<tr><th>J</th><th>F</th><th>M</th><th>A</th><th>M</th><th>J</th><th>J</th><th>A</th><th>S</th><th>O</th><th>N</th><th>D</th><th>J</th><th>F</th><th>M</th><th>A</th><th>M</th><th>J</th><th>J</th><th>A</th><th>S</th><th>O</th><th>N</th><th>D</th></tr>
<tr><td colspan="12">LCS 2 CONUS</td><td colspan="12">LCS 2 Deployed</td></tr>
<tr><td colspan="4">CREW 201</td><td colspan="4">CREW 202</td><td colspan="4">CREW 203</td><td colspan="4">CREW 203</td><td colspan="4">CREW 201</td><td colspan="4">CREW 202</td></tr>
<tr><td colspan="12">LCS 4 Deployed</td><td colspan="12">LCS 4 CONUS</td></tr>
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<tr><td colspan="24">OFF-HULL</td></tr>
<tr><td colspan="4">CREW 202</td><td colspan="4">CREW 203</td><td colspan="4">CREW 201</td><td colspan="4">CREW 202</td><td colspan="4">CREW 203</td><td colspan="4">CREW 201</td></tr>
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Figure 4. Steady-state crew rotation concept

Crew rotation should occur every four months. A single crew begins its typical flow in San Diego and not assigned to any of the LCS ships. This is referred to as an “Off-Hull” period. Here the crew uses shore-based training devices to attain their initial qualification in preparation for deployment. This training is under the guidance of the LCSRON training department. Once LCSRON designates a crew as qualified they transition onto the CONUS-based LCS in their pair. This is referred to as the “On-Hull” period. During the On-Hull period the crew maintains their proficiency by operating the CONUS hull performing training and missions supporting the Commander of U.S. Third Fleet. Lastly, the crew will rotate to the deployed hull to conduct missions supporting the combatant commander in theater.

USS *Independence* was commissioned in January 2010. At that time there were two LCS crews (Crew 201 and Crew 202) assigned to the single ship. They operated in a Blue-Gold rotation plan similar to how U.S. Navy submarines have operated for decades. In April 2014, USS *Coronado* was commissioned and the third crew in the pair (Crew 203) was established. Crew 201 and Crew 202 were highly experienced having made numerous deployments over the past four years; however Crew 203 had not completed a full training and deployment cycle. LCSRON assigns Crew 203 to USS *Coronado* during Builder's Trials, but has a lot of flexibility to choose another experienced crew for the CMA phase.

The LCS Planning Schedule, as of October 2013, calls for the construction of 24 LCS ships. Instead of having years between the first ship in a pair and the second ship in a pair, future LCS will be staggered by only five or six months. The first disruption to the 3-2-1 rotation plan occurs when the first ship in a new pair begins the CMA phase. Since the two crews established for this new ship have not deployed, a crew from outside the pairing must be selected. Additionally, since the ships are now being produced faster, it does not allow either of the two newly established crews to complete a deployment cycle and have the experience for the CMA phase of the second ship in the pair.

The LCSRON Commander is willing to accept some disruption in the core concept of the 3-2-1 plan in order to have an experienced crew on a ship during the CMA phase. However, long-term disruption of the crew-ship pairings is not desired. Maintaining the integrity of the crew-ship pairing has a long-term effect of reducing the time required to conduct a proper turnover between crews.

C. OBJECTIVES

LCSRON assigns crews to ships manually. This method will not necessarily find the optimal crew allocation that minimizes the disruption to the 3-2-1 rotation plan. The goal of this thesis is to develop LCSS, a mathematical optimization model to aid LCSRON's decision making process of assigning LCS crews to LCS ships. LCSS's objective is to minimize the penalty cost of assigning a crew to a ship outside of its designated pair while satisfying the required training flow prior to operational deployments and experience requirements of new ship construction. LCSS is designed to guide the scheduler by producing an initial quality solution that can be modified to account for other intangible requirements of the command.

D. THESIS SCOPE AND ORGANIZATION

A crew is considered a single entity for the purposes of LCSS. The USN rotates sailors from sea-duty commands to shore-duty commands in order to satisfy the mandated sea-shore rotation of all naval personnel. This can lead to different manning levels on different LCSs as time progresses. LCSS assumes that each crew is sufficiently manned to accomplish the mission to which it has been assigned at any given time period.

LCSS solves (as independent problems) the schedule for the Freedom and Independence variants of LCS. However, for the purposes of this thesis, only the Independence variant is presented.

While the primary LCS homeport is San Diego, California, there are future plans for some LCSs to be stationed in Mayport, Florida. There are personnel and accounting challenges beyond the scope of this thesis that make it infeasible to use San Diego crews as experienced crews for Mayport ships. LCSS (in its current form) is not intended to solve that problem, but paves the road for an extended version of the optimization model that takes those considerations into account.

This thesis has five chapters: Introduction, Literature Review, Model Development, Model Implementation and Conclusions. In Chapter II, we discuss the field of scheduling optimization. This includes common techniques to solve optimization problems, heuristic techniques to reduce solve time and previous thesis research in the field of military scheduling. Chapter III discusses the development of LCSS as a mixed-integer, linear model which captures the scheduling specifications of LCSRON. This chapter also discusses the use of three methods (one exact and two heuristic) to solve

LCSS. In Chapter IV, LCSS results are presented, and a realistic impact of the solution is discussed. Also, the three methods are compared for solve time and solution quality. Finally, Chapter V contains the conclusions and suggests future work that could benefit LCSS.

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II. LITERATURE REVIEW

Most scheduling optimization problems fall into the broad class of discrete optimization models known MIPs, and are typically difficult to solve computationally. LCSS falls within this category.

Rardin (1998) discusses some methods to solve MIPs. One approach is through total enumeration. However, practical problems usually have too many solutions to evaluate. “Branch and Bound” uses linear relaxation and select partitions of the feasible region. It identifies search avenues to find integer solutions while avoiding expending resources on exploring regions that cannot contain an optimal solution. This method still does not succeed in reducing solve times to a practical level on many real-world applications, but it does allow for a solution to a provable level of optimality, at least in theory.

Rardin (1998) further discusses a range of scheduling applications. Some are based on set packing and set partitioning models which use “mutual[ly] exclusive constraints involving subsets of decision variables” (Rardin, 1998, p. 566). For example, one such model assigns aircrews to a commercial airline in order to minimize cost while ensuring every flight is flown.

Airlines are conscious of the cost of crew assignment but also desire a robust schedule that does not incur delays and disruptions to future flights because of near-term changes. Ehrgott and Ryan (2003) use set partitioning to solve a crew scheduling problem for commercial airlines using bicriteria optimization balancing operating costs and robust scheduling.

Rardin (1998) also describes the job-shop scheduling model. This model seeks an optimal allocation of a set of tasks to a set of machines to create a product. A task may have restrictions on its start time and may require multiple machines in a specified sequence. Further, machines may have their own restrictions on operation and which tasks can be performed in succession. These restrictions generate the flow through the system using a transition matrix that defines a set of valid machines a product can visit next.

Job-shop scheduling models can be applied to DON scheduling processes. Goodman (1985) uses this to assign Atlantic Fleet surface combatants to fulfill commitments at home and abroad. This approach categorizes the requirements of the Atlantic fleet into discrete events with fixed start and stop time. Real-world operations and major exercises are classified as primary events. Major maintenance and operations that are necessary to support the completion of primary events are also scheduled but with a lower priority than primary events. Units are separated into functional categories so that units with similar operational capabilities are grouped. The Combatant Primary Event Schedule then matches units to events while distributing the workload across all units in an equitable manner.

Farmer (1992) uses optimization to assign U.S. Coast Guard cutters to the First Coast Guard District. The goal is to improve the response capability for coastal search and rescue as well as law enforcement tasking. Cutters are limited on how long they can be in a ready status, while the district has requirements to cover a defined patrol area. Additionally, it is desirable to make assignments in an equitable manner to maintain materiel condition and crew proficiency. The Cutter Scheduler generates a quarterly schedule by week to satisfy the requirements of the Coast Guard District.

Madson (2010) develops a Carrier Optimal Strike-Fighter Scheduling Tool using a time-phased resource allocation approach. This model assigns strike-fighter squadrons to carrier airwings to support the Navy's Fleet Response Plan. Since there are not enough strike-fighter squadrons available to fill every carrier airwing, it is necessary to move squadrons between airwings. These moves are assigned in a manner that provides full combat capability to deploying carrier airwings while minimizing the number of moves each squadron makes. Minimizing the number of moves is desired because each move incurs a monetary cost. Also, the operational tempo of each squadron needs to be managed within prescribed naval regulations

Jacobs (2014) develops a Flight Training Scheduler to pair instructor pilots with student aviators to maximize the number of syllabus events that a squadron generates each day. This model assigns instructors with specific qualifications to students with specific requirements to available aircraft during a single day of training.

The LCS scheduling problem takes on attributes of both of these model classes: time-phased resource allocation and job-shop scheduling. Each crew is akin to the aircrew in the airline schedule model and must be assigned to each ship to conduct a mission. LCSS is similar to a job-shop scheduling problem where the ship takes the role of the product and the crew is the machine. In this way, LCSS is assigning crews to ships (i.e., “machines” to “products”), where these have given time windows (known as ship phases, such as “Deployed” or “On-conus”). Assignments must satisfy both the ship schedule and the crew transitions. Crew phase transitions have similar connotations to precedence relationships in job-shop scheduling. Interestingly, LCSS must apply them to the “machine” (crew) as it evolves (for example, from phases not requiring experience to phases requiring it), instead of to the “product” (ship).

To further reduce solve times for practical application there are useful heuristic approaches. Sethi and Sorger (1991) discuss how the business aspect of optimization can rarely forecast future issues with certainty. They apply a rolling horizon (RH) approach which optimizes over a shorter time period, ignoring future events. This time period is then rolled into the future and the model is solved again with previously determined variables fixed by the earlier solution. While this heuristic succeeds in reducing total solve time, it is not guaranteed to find an optimal, or even feasible, solution for the long-term schedule because decisions made in earlier time periods may be irreversible in the future. LCSS is solved using this RH technique for faster solutions, as described in Section III.B.1.

To mitigate the myopic view of RH, Dillenberger et al. (1994) introduce Fix and Relax (F&R) in which instead of ignoring future events, associated variables are relaxed to be continuous. Similar to RH, F&R suffers from lack of optimality guarantee, but the relaxed models take the future schedule into consideration, which intuitively should help improve the RH result.

Escudero and Salmeron (2005) discuss an implementation of F&R for project selection and scheduling. Their method solves an integer assignment problem in multiple phases by retaining integer conditions for a subset of decision variables. The peculiarity here is that the grouping of variables that produces the best results is not necessarily associated with time periods. Our LCS model also uses F&R as described in Section III.B.2.

In practice, assignment problems are generally not one-time occurrences, but must be repeated periodically or even adjusted as circumstances change. Brown, Dell and Wood (1997) discuss the importance of persistence in dynamic assignment problems. They cite a shipping company that has packages sorted in trucks to optimally route them to their destination. When one additional package arrives, the optimal solution could change dramatically. However, the company does not desire to unload every truck and repack it with the new optimal solution since this will cost too much time and money. Instead, the model should take into account the current state of the system and determine an optimal solution, within determined bounds, that minimizes the number of changes required to the current system.

A time-based resource allocation problem with persistence can be implemented in two phases using separate models. Pickett (2013) develops two models to schedule USN submarine tenders. The first model assigns workers to tasks over time given planned maintenance demands, job precedence's, time windows, and other constraints. Then, the second model takes that schedule and adjusts it to minimize changes in response to demand updates.

We have not implemented persistence in LCSS, but considering the potential changes in ship production schedules, we deem it would be a useful extension of this work for the operational planner.

III. MODEL DEVELOPMENT

A. LITTORAL COMBAT SHIP SCHEDULER

The LCSS is a MIP that assigns LCS crews to LCS ships over a given time horizon (e.g., the next three years) as the fleet accepts new LCS hulls and new LCS crews into the inventory. The LCS Planning Schedule (LCS Program Manager, 2013) is generated by the LCS Program Office and specifies the Navy's schedule for the construction of new LCS hulls. This schedule is promulgated to Congress for budgetary purposes and to the contractors involved in the LCS program for shipyard scheduling. LCSRON also maintains a long-range schedule of operational commitments, deployments and periodic maintenance for its ships. The current LCS Long Range Schedule and the LCS Planning Schedule (LCS Program Manager, 2013) are used as inputs to the LCSS model. LCSS assigns penalties to each crew-ship pair based upon whether or not the crew is supposed to be primarily assigned to the ship. LCSS then minimizes the total penalty in order to encourage crews to remain within their assigned ship pairing, while still accomplishing the mission objectives of LCSRON.

1. Problem Specifications

The time horizon is divided into monthly time periods. However, not all crews and ships exist at every period. The model is instantiated at "time zero" with only a subset of crews and ships in existence. These crews have been assigned to ships for a known number of periods. Also, some of these crews have already attained the qualification of "experienced" through previous deployments. As time progresses, new crews and ships are available for scheduling as prescribed in the LCS Planning Schedule (LCS Program Manager, 2013). These are accounted for as input to LCSS with the time period a crew or ship can first be assigned by LCSS. The new crews are not experienced when they are formed and therefore can only be assigned to a subset of ship missions.

Ship missions are separated into six phases: Construction, "Precom", Acceptance, "CONUS-Off" (fictitious, see below), "CONUS-On," and Deploy. The Construction phase covers all aspects of ship building dictated by the LCS Program manager from the award date of the contract until the ship's christening. Once Builder's Trials begin the ship enters the Precom phase which lasts the duration of the trials. This phase requires a crew to be assigned to the ship, but these sailors do not need to be experienced. Since at least one new crew is procured specifically for the ship entering the Precom phase it is logical that this crew will assume those duties, but LCSS is not constrained to that assignment. The CMA milestone signals the beginning of the Acceptance phase. At this point, a crew with deployment experience is required to be assigned to the ship until it arrives in its homeport. At that point, the assignment of its active phase transitions from the Shipbuilding Plan to the Long Range Schedule. The ship is now restricted to one of two phases: CONUS-On if it is located near San Diego for tasking by Third Fleet, or Deploy if it is forward deployed to Singapore.

LCS ship phases for each time period are inputs to LCSS. Each LCS crew assumes the phase of the ship to which it is assigned in a given time period. Remark: A fictitious CONUS-Off phase is reserved for "dummy-ships" created as assignment locations for the third crew in the pair that is conducting training in San Diego. No actual LCS will be in the CONUS-Off phase but this fictitious phase allows for the desired crew rotation plan to be mathematically implemented in LCSS.

Crews are restricted to a subset of allowed phase transitions during any period (see Table 2). For example, crews cannot be assigned to the Construction phase preventing assignment prior to Builder's Trials (Precom). Crews are also prevented from transitioning from a CONUS-Off phase to a Deployed phase because of the required certification process during the CONUS-On phase. However, LCSS is flexible to accommodate changes in user-allowed transitions.

Crews are assigned to ships for a minimum number of time periods in a particular phase (which is determined by the LCS Commander), currently four months. However, there is leeway for elasticity to accommodate the competing requirements of crew-ship pairing integrity and acceptance experience up to a maximum of six months. Extending in a given phase is not desired, therefore it incurs a penalty in LCSS.

Table 2. Valid crew-phase transitions

	Precom	Accept	CONUS-On	CONUS-Off	Deploy
Precom	YES	NO	YES	YES	NO
Accept	NO	YES	YES	YES	YES
CONUS-On	NO	YES	YES	YES	YES
CONUS-Off	YES	YES	YES	YES	NO
Deploy	NO	YES	NO	YES	YES

2. Assumptions

LCSS makes the following assumptions:

a. Crew Entities. Each crew is treated as a complete, single unit. The individual manning levels for each crew are assumed to be sufficient to carry out all assigned missions.

b. Attaining Experience. A crew can only attain the flag “Experienced” from being assigned to a ship on deployment. Therefore, a crew that accomplishes multiple workup cycles without being deployed will never be assigned to the Acceptance phase even though it is arguable that they would be capable of that mission after a long enough period of time in existence.

c. Persistent Experience. Once a crew is designated as experienced it will always be considered experienced. Periodic rotation of individuals will occur according to the Navy Personnel Command guidance, but sailors will have longevity in the LCS platform so that leadership positions will have enough experienced personnel assigned.

d. Fixed Ship Schedule. The schedule for each ship is assumed to be predetermined. LCSS does not suggest any changes to ship schedules, even if those could improve the overall assignment of crews to LCS hulls.

3. Littoral Combat Ship Scheduler Formulation

Indices and Sets

$c \in C$	set of LCS crews {crew201, crew202, crew203,…}
$s \in S$	set of LCS hulls {LCS2, LCS4, LCS6,…}
$p \in P$	set of ship phases {Construction, Precom, Accept, CONUS-Off, CONUS-On, Deploy}
$t \in T$	set of time periods {0,1,2,…} [months] (indexed set)
$t_0^c \in T$	time period in which crew *c* becomes available to schedule
$t_0^s \in T$	time period in which ship *s* becomes available to schedule
TR	subset of pairs (*p*,*p'*) where a transition from phase *p* to phase *p'* is valid
$Q \subset S \times P \times T$	subset of triplets (*s*,*p*,*t*) where ship *s* is in phase *p* at time *t*

Data

$AssignCost_{c,s}$	cost associated in assigning crew *c* to ship *s* [penalty units]
$InitialPair_{c,s}$	one if crew *c* is assigned to ship *s* at time 0, zero otherwise
$InitialExp_{c,t}$	one if crew *c* is experienced at time 0, zero otherwise
$PreExist_c$	number of time periods crew *c* has been assigned to the current ship at time 0. [months]
minLength	minimum time a crew can remain in a phase [months]
ρ_1	penalty for extending a crew one additional month [penalty units]
ρ_2	penalty for extending a crew two additional months [penalty units]
$\Delta_{t,t'}$	one if $t = t'$, zero otherwise

Derived Sets

$T_c^C \subset T$ subset of time periods in which crew c is available for scheduling.

Calculated as $T_c^C = \{t \in T \mid t \geq t_0^c\}$

$T_s^S \subset T$ subset of time periods in which ship s is available for scheduling.

Calculated as $T_s^S = \{t \in T \mid t \geq t_0^s\}$

Binary Variables

$X_{c,s,t}$ one if crew c is assigned to ship s at time t

$Y_{c,s,t}$ one if crew c starts on ship s during time t

$E_{c,t}$ one if crew c is considered experienced at time t

$H_{c,p,t}$ one if crew c is in phase p at time t

$A_{c,p,t}$ one if crew c starts in phase p at time t

$D1_{c,t}$ one if crew c extends one additional month in any phase at time t

$D2_{c,t}$ one if crew c extends two additional months in any phase at time t

Formulation

$$\text{Minimize } \sum_{c,s,t} AssignCost_{c,s} X_{c,s,t} + \sum_{c,t} (\rho_1 D1_{c,t} + \rho_2 D2_{c,t}) \tag{1}$$

Subject to:

$$\sum_{c \mid t \in T_c^C} X_{c,s,t} = 1 \quad \forall s,t \mid t \in T_s^S \tag{2}$$

$$\sum_{s \mid t \in T_s^S} X_{c,s,t} = 1 \quad \forall c,t \mid t \in T_c^C \tag{3}$$

$$Y_{c,s,t} \geq \sum_{s' \mid s' \neq s} X_{c,s',t-1} + X_{c,s,t} - 1 \quad \forall c,s,t \mid t \in T_c^C \tag{4}$$

$$\sum_{t'|t\le t'\le t+6} H_{c,p,t'} \le minLength - (PreExist_c \Delta_{t,0}) + D1_{c,t} + D2_{c,t} \quad \forall c,p,t \tag{5}$$

$$A_{c,p,t} \le H_{c,p,t'} \quad \forall c,p,t,t' \,|\, t \ne 0 \;\wedge\; t \le t' \le t + minLength \tag{6}$$

$$A_{c,p,0} \le H_{c,p,t'} \quad \forall c,p,t' \,|\, t \le t' \le t + minLength - Prexist_c \tag{7}$$

$$Y_{c,s,t} \le X_{c,s,t'} \quad \forall c,s,t,t' \,|\, t \in T_c^C \cap T_s^S \setminus \{0\} \;\wedge\; t \le t' \le t + minLength \tag{8}$$

$$Y_{c,s,0} \le X_{c,s,t'} \quad \forall c,s,t' \,|\, t \in T_c^C \cap T_s^S \wedge t \le t' \le t + minLength - Prexist_c \tag{9}$$

$$Y_{c,s,t} \le X_{c,s,t} \quad \forall c,s,t \,|\, t \in T_c^C \cap T_s^S \tag{10}$$

$$X_{c,s,0} = InitialPair_{c,s} \quad \forall c,s \tag{11}$$

$$Y_{c,s,t_0^c} \ge X_{c,s,t_0^c} \quad \forall c,s \,|\, t_0^c \in T_s^S \setminus \{0\} \tag{12}$$

$$Y_{c,s,0} \ge X_{c,s,0} \quad \forall c,s \,|\, PreExist_c = 0 \;\wedge\; \{0\} \in T_s^S \tag{13}$$

$$E_{c,0} = InitialExp_{c,0} \quad \forall c \tag{14}$$

$$\sum_{s|(s,p,t)\in Q} X_{c,s,t} = H_{c,p,t} \quad \forall c,p,t \,|\, t \in T_c^C \tag{15}$$

$$E_{c,t} = 0 \quad \forall c,t \,|\, t \notin T_c^C \tag{16}$$

$$E_{c,t} \le E_{c,t-1} + \sum_{s|(s,"Deploy",t)\in Q} X_{c,s,t} \quad \forall c,t \,|\, t \in T_c^C \setminus \{0\} \tag{17}$$

$$E_{c,t} \ge E_{c,t-1} \quad \forall c,t \,|\, t \in T_c^C \setminus \{0\} \tag{18}$$

$$E_{c,t} \ge \sum_{s|(s,"Deploy",t)\in Q} X_{c,s,t} \quad \forall c,t \,|\, t \in T_c^C \setminus \{0\} \tag{19}$$

$$X_{c,s,t} \le E_{c,t} \quad \forall c,s,t \,|\, t \in T_c^C \cap T_c^S, (s,"Accept",t) \in Q \tag{20}$$

$$X_{c,s,t} = X_{c,s,t-1} \quad \forall c,s,t \,|\, t \in T_c^C \cap T_c^S \;\wedge\; (s,"Accept",t) \in Q \;\wedge\; (s,"Accept",t-1) \in Q \tag{21}$$

$$X_{c,s,t} = X_{c,s,t-1} \quad \forall c,s,t \mid t \in T_c^C \cap T_c^S \wedge (s,"Precom",t) \in Q \wedge (s,"Precom",t-1) \in Q \tag{22}$$

$$A_{c,p,t} \geq \left(\sum_{p' \mid p' \neq p} H_{c,p',t-1} \right) + H_{c,p,t} - 1 \quad \forall c,p,t \mid t \in T_c^C \setminus \{0\} \tag{23}$$

$$1 - \sum_{p} A_{c,p,t} + D1_{c,t} \Delta_{t',t-4} + D2_{c,t} \Delta_{t',t-5} \geq \sum_{s \mid t' \in T_s^S} Y_{c,s,t'}$$

$$\forall c,t,t' \mid t \in T_c^C \wedge t \leq t' \leq t + minLength + 2 \tag{24}$$

$$\sum_{p' \mid (p',p) \in TR} H_{c,p',t-1} \geq H_{c,p,t} \quad \forall c,p,t \mid t-1 \in T_c^C \tag{25}$$

$$X_{c,s,t} = 0 \quad \forall c,s,t \mid t \notin T_c^C \cap T_s^S \tag{26}$$

$$H_{c,p,t_0^c} = 0 \quad \forall c,p \mid p \in \{Construction, Accept, Deploy\}, t_0^c \in T_c^C \setminus \{0\} \tag{27}$$

$$X_{c,s,t}, Y_{c,s,t}, E_{c,t}, H_{c,p,t}, A_{c,p,t}, D1_{c,t}, D2_{c,t} \in \{0,1\} \quad \forall c,p,s,t \tag{28}$$

4. Explanation of Formulation

Equation (1) defines the objective function which expresses the total penalty of the crew scheduling assignment. The first part expresses the penalty of assigning a crew to a ship. The second part expresses the penalty of a crew remaining in the same phase for more than the minimum length of consecutive time periods.

The objective function is driven by the tradeoffs between keeping crews in their assigned pairing and the penalty cost of extension, ρ_1 and ρ_2. If extending crews is highly undesirable, the extension penalties (ρ_1 and ρ_2) should be set to a large number that outweighs the assignment cost, $AssignCost_{c,s}$.

Equation (2) requires that one crew is assigned to every ship that is in operation at each time period. Equation (3) requires that one ship is assigned to every crew that has been established during each time period. Equation (4) flags the shift of a crew to a new ship during successive time periods.

Equation (5) controls the maximum number of time periods a crew can remain in a particular ship phase (up to the minimum number plus two months). This equation includes data on the pre-existing number of months a crew has been assigned to a ship, $PreExist_c$, before the model start point. It also contains the two elastic variables, $D1_{c,t}$ and $D2_{c,t}$, that control extending a crew in a phase more than the desired number of months.

Equations (6) and (7) control the minimum number of time periods that a crew can remain in a particular ship phase. Equation (7) handles special case of time zero, where a crew has already been in a particular phase for some time.

Equation (8) and (9) control the minimum number a months a crew must be assigned to a ship before they are allowed to rotate to a new ship. Equation (9) accounts for the time periods a crew has already been assigned to a ship before time zero.

Equation (10) ensures that crews are assigned to the ship on which they have begun a phase.

Equations (11)–(14) establish the initial conditions LCSS. They determine the initial crew-ship pairings and the initial experience level of all crews.

Equation (15) assigns a crew to the same phase as the ship to which the crew is assigned.

Equation (16)–(19) handle the experience of each crew. Equation (16) ensures that crews that are not in existence cannot be experienced. This prevents LCSS from designating a crew as experienced at the time they are established. A crew becomes experienced during each time period if it was previously experienced or it is currently on a ship that is in a deployed phase.

Equations (20) and (21) validate that only an experienced crew is assigned to a ship in the Acceptance phase of its acquisition. It also prevents a crew from being assigned to a different ship while the current ship remains in the Acceptance phase.

Equation (22) ensures the continuity of crew assignment during the pre-commissioning phase of acquisition.

Equation (23) identifies when a crew changes phases.

Equation (24) prevents a crew from changing phases until after the minimum number of time periods have elapsed plus any additional time periods as indicated by the elastic variables $D1_{c,t}$ and $D2_{c,t}$.

Equation (25) ensures that a crew can only make valid transitions between two phases (see Table 2).

Equation (26) sets the value of the crew assignment variable to zero when they crew is not in existence.

Equation (27) prevents a crew that has just been established from being assigned to a ship in the Acceptance or Deployed phase because they do not have the required training for those missions at that time.

Equation (28) controls the decision variables' domains as binary.

B. HEURISTIC SIMPLIFICATION

The LCS Shipbuilding plan for the Independence and Freedom LCSs involve building twelve ships with eighteen funded crews over the next five years for each variant. That makes the LCSS formulation computationally intractable (see Chapter IV) even for an advanced commercial MIP solver. Therefore, it is imperative to explore heuristic methods to generate feasible solutions.

The schedule of an individual LCS is impacted by numerous external events. The day-to-day job of the operational planner involves making changes to the short and long-range ship schedule to account for operational requirements, maintenance casualties or shipyard delays. Short-term changes in input data have an immediate impact on the optimal solution for LCSS. Also, due to uncertainty in long-term input data, LCSS should weight the short-term schedule more than the long-term schedule.

1. Rolling Horizon

In this section, we discuss the implementation of RH (Sethi & Sorger, 1991) to address the solve time challenges involved with solving the full LCSS MIP. A RH solution establishes a hierarchy favoring short-term over long-term schedules.

Implementing RH requires establishing the number of time horizons, here on referred to as sub-problems, *n*, to be solved. Each sub-problem, *sp*, must have a defined length, *l*, of time periods. Within each sub-problem there is a subset of time periods where variables are fixed, referred to as the *fixed period*, and time periods that are not fixed, referred to as the *unfixed period, u*. The first sub-problem (*sp*=1) has no fixed period, but subsequent sub-problems have fixed and unfixed periods. Successive sub-problems have overlapping unfixed periods. These are referred to as the *unfixed overlap, d*.

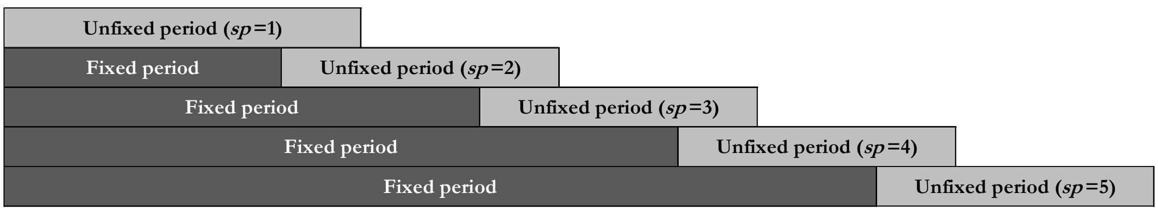

Figure 5. Depiction of rolling horizon with unfixed overlapping time periods

Figure 5 graphically depicts the RH process. The first sub-problem (*sp*=1) has the smallest time horizon and must be solved feasibly. If no feasible solution is returned then LCSS is infeasible.

The feasible solution, excepting the unfixed overlap, is set as fixed data for the second sub-problem (*sp*=2). This process continues until all sub-problems have been solved and return a feasible solution. Infeasibility for any sub-problem (except the first one) does not mean the original problem is infeasible, but does illustrate the pitfall of a myopic RH process.

The choice of *n* drives how quickly a feasible solution is generated. If *n* is small, it intuitively forces *l* and *u* to be large. Since the size of the unfixed period, *u*, governs solve times, this will result in unacceptably long times. On the other hand, if *n* is too large and *u* is too small, LCSS will be assigning assets without consideration for future events and schedules.

The selection of *u* for each sub-problem also has the same tradeoff considerations of balancing solve time and overly myopic scheduling. However, the unfixed period length for each sub-problem does not need to be the same. The structure of the LCS Shipbuilding Schedule (see Figure 6) introduces new ships and new crews as time progresses. This means that later time periods will have significantly more decision variables to consider. The unfixed period for the first sub-problem is selected to be as long as possible while still returning a solution in an acceptable time for the planner (e.g. one day). Subsequent time horizons are shortened to balance their solve times to make the full solution available to the operational planner in a reasonable time.

	t0	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11	t12
LCS2	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On
LCS4	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On
LCSa	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off
LCS6	Accept	Accept	Accept	Accept	Accept	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On
LCS8	Precom	Precom	Precom	Precom	Precom	Accept	Accept	Accept	Accept	Accept	CONUS-On	CONUS-On	CONUS-On
LCSb	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off
LCS10							Precom	Precom	Precom	Precom	Precom	Accept	Accept
LCS12													Precom
LCSc							CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off
LCS14													
LCS16													
LCSd													
LCS18													
LCS20													
LCSe													
LCS22													
LCS24													
LCSf													

Figure 6. Partial LCS Ship Schedule

The myopic view of RH can lead to sub-optimal assignment of crews near the end of each time horizon (a.k.a. "end effects"). Thus, our implementation of RH relaxes the boundary conditions between successive sub-problems using the unfixed overlap, *d*: if a crew assignment made by the previous sub-problem falls in the unfixed overlap, the subsequent sub-problem can modify that assignment. This allows for better crew assignments, at least in theory.

A time horizon must be defined for each sub-problem $T_{sp}^{RH} \subset T$. This begins at time zero for each sub-problem, monotonically increases until all time periods are considered for the final sub-problem.

The MIP formulation is modified to implement RH. For example, the objective function (1) is modified to the form of Equation (28):

$$\sum_{c,s,t|t\in T_{sp}^{RH}} AssignCost_{c,s} X_{c,s,t} + \sum_{c,t|t\in T_{sp}^{RH}} (\rho_1 D1_{c,t} + \rho_2 D2_{c,t}) \tag{28}$$

where the t index is now restricted to a subset of periods T_{sp}^{RH} related to sub-problem sp. We also add constraints to fix variables in the fixed period.

2. Fix and Relax

In this section, we discuss the implementation of F&R (Dillenberger, 1994) to partially mitigate the myopic approach of RH. F&R implements a sequence of mixed-0–1 sub-problems (see Figure 7). F&R allows the model to consider future ship schedules to generate a better crew-ship assignment. For each sub-problem, the F&R implementation requires explicit definition of three sets: fixed decision variables (T_{sp}^{Fixed}), binary decision variables ($T_{sp}^{Integer}$) and continuous decision variables ($T_{sp}^{Continuous}$). Once again, an unfixed overlap period is employed to allow the model to make changes to the last periods in the preceding sub-problem.

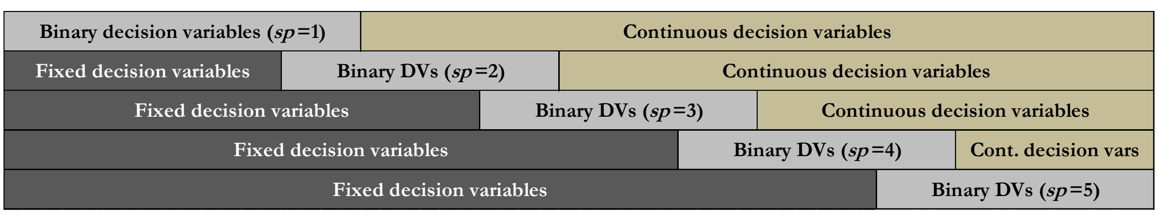

Figure 7. Depiction of F&R with unfixed overlapping time periods

Each decision variable in the formulation must be modified to its appropriate type during each sub-problem. For example, in Equation (2), implementing F&R results in the new Equation (28) which incorporates three decision variables but controls the sub-problems in which the model uses them:

$$\sum_{c|t\in T_c^C \cap T_{sp}^{Fixed}} X_{c,s,t}^{Fixed} + \sum_{c|t\in T_c^C \cap T_{sp}^{Integer}} X_{c,s,t}^{Integer} + \sum_{c|t\in T_c^C \cap T_{sp}^{Continuos}} X_{c,s,t}^{Continuous} = 1 \quad \forall s,t \,|\, t \in T_s^S \tag{28}$$

The binary variables in vectors X, Y, H and A are substituted by original, continuous or fixed versions of the variables. In this manner, the lifetime of a decision variable begins as continuous, then becomes integer and finally becomes fixed. The “Fixed” version holds the integer solution from previous sub-problems. The “Integer” version behaves in the same manner as the original variable in the MIP. The “Continuous” version is the main distinction between F&R and RH. F&R uses fractional values to satisfy future model conditions. This reduces the fully myopic aspect of RH where future requirements are completely oblivious to the incumbent sub-problem.

Selecting the size of $T^{Integer}$ determines the Fixed and Continuous subsets for each sub-problem. The Integer subset size drives the solve time, but the size of $T^{Continuous}$ is also important. Since all time periods are considered in each sub-problem, solve times will generally be longer than for RH.

Figure 7 graphically depicts LCSS’s implementation of F&R. During the first sub-problem all variables are either $T_1^{Integer}$ or $T_1^{Continuous}$. The resulting integer values in the $T_1^{Integer}$ subset are used as the values for T_2^{Fixed} in the second sub-problem except for those decision variables in the unfixed overlap period. The second stage is then solved. If the model is infeasible during any sub-problem the process stops and the selection of subset size and solve parameters must be adjusted. The process continues until the last sub-problem, consisting of only fixed and integer variables, is solved.

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IV. MODEL IMPLEMENTATION

LCSS is implemented with the General Algebraic Modeling System (GAMS) using the GAMS/CPLEX (GAMS, 2014) solver, on a Dell Precision T7500 computer running at 3 GHz with 48 Gb of RAM. The model contains approximately 140,000 equations and 700,000 binary variables. The three solving approaches discussed in Chapter III generate different scheduling solutions.

First, the MIP formulation is solved for different time horizons as separate runs: 12, 15,…,30 months. A desirable goal would be to generate a solution for up to three years, but the computational time for the MIP formulation would be unacceptable. Outputs are compared to each other to assess if LCSS selects the same crew-ship assignments. This would indicate if solutions are nested, that is, if shorter-term solutions are independent of future events. Next, RH and F&R are used to solve a 40-month horizon. The idea here is to analyze the solution for the first 36 months, but plan for 40 months to partially avoid end effects. Their solutions are compared to the longest (30-month) MIP solution available. The objective function, Z, returned by LCSS is used to assess how well the solution keeps crews in their desired pairing. For comparison in this thesis, the Z values are specified by the method used and full time horizon of the method. For example, the 30-month MIP objective value is represented as $Z^{MIP,30}$.

All three approaches use the following penalties: The *AssignCost* penalty is zero for crews assigned to their desired ship pairings or fictitious ship (see Table 3). Crews assigned to an actual ship outside their desired pairing are assessed a penalty of ten, and crews assigned to a fictitious ship (e.g., LCS "b") outside of their desired pairing are assessed a penalty of 1,000. If LCSS makes an out-of-pair assignment to CONUS-Off (a fictitious ship) then it is difficult to keep crews in their desired training cycl—CONUS-Off, CONUS-On, Deploy—as this would put two crews in CONUS-Off at the same time, which would require additional schedule disruption to re-establish the desired flow. Additionally, ρ_1 and ρ_2 are set to one and two, respectively. This favors keeping crews in their assigned ship pairing over extending crews in phase. However because of the interactions between crews, changing the values of ρ_1 and ρ_2 can lead to different LCSS solutions.

Table 3. Desired LCSRON crew-ship pairings

LCS 2/LCS 4 (LCS a)	LCS 6/LCS 8 (LCS b)	LCS 10/LCS 12 (LCS c)	LCS 14/LCS 16 (LCS d)	LCS 18/LCS 20 (LCS e)
Crew 201	Crew 204	Crew 207	Crew 210	Crew 213
Crew 202	Crew 205	Crew 208	Crew 211	Crew 214
Crew 203	Crew 206	Crew 209	Crew 212	Crew 215

A. MIXED-INTEGER LINEAR PROGRAM IMPLEMENTATION

1. Solution Comparisons

CPLEX can solve the linear relaxation of the 40-time period LCSS model, but is unable to obtain an integer, feasible solution after searching for over 48 hours. Therefore, we solve the MIP for reduced time horizons of 12, 15,…, 30 months and compare them to assess if there is any evidence of nested solutions. A nested solution occurs when the crew assignments for a shorter time horizon model (e.g., 12 months) are the same as for a longer-term model (e.g., 21 months). All instances are solved to a 5% relative optimality.

Figure 8 shows a comparison of the solve times required for each case. It is apparent that solve times increase exponentially.

We explore if the solutions are nested by first comparing the objective value of successive MIP runs. Table 4 summarizes the results of LCSS for each MIP horizon. The objective value and number of crew assignments (CA) made by LCSS are shown. Then the change in objective value per change in crew assignments ($\Delta Z/\Delta CA$) for successive time periods is calculated. If this number is the same for successive periods it is an indication that solution nesting may exist. The values of $\Delta Z/\Delta CA$ suggest there may be nesting, but we must conduct more exploration.

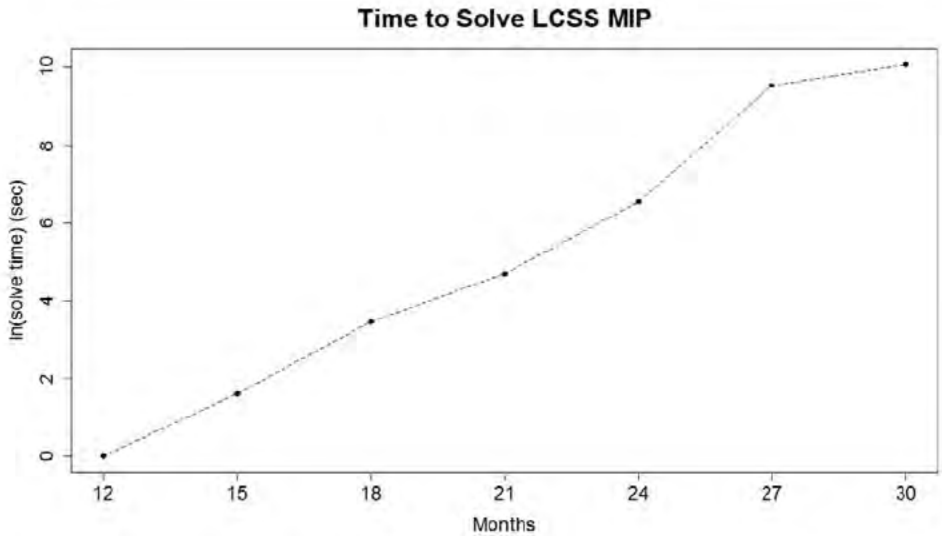

Figure 8. LCSS solve times in ln(seconds) for varying time horizons

The 12-month schedule has initial conditions that force crews to remain on ships for a minimum of four months. LCSS only has the opportunity to make one or two crew swaps per ship; these swaps do not consider assigning a crew to the Acceptance phase. Therefore, the 12-month MIP is a simple task of assigning crews to ships akin to the long-term steady state concept.

Table 4. Summary of objective value, crew assignments and average penalty for LCSS MIP

Months	**Objective value ($Z^{MIP,months}$)**	**Crew Assignments**	**Change in Objective Value (ΔZ)**	**Change in Crew Assignments (ΔCA)**	**$\Delta Z / \Delta CA$**
12	1,279	84	--	--	--
15	4,338	111	3,059	27	113.0
18	4,434	138	96	27	3.5
21	4,549	171	115	33	3.5
24	4,648	204	99	33	3.0
27	4,952	240	304	36	8.4
30	5,004	276	52	36	1.4

Next, we continue our assessment of nesting by comparing the assignments made by LCSS for every pair of solutions during the time horizon of the shorter model. For example, when comparing the 12-month and 30-month MIP solutions we only consider the first 12 time periods. LCSS-generated schedules for 12-month MIP and the first 12 months of the 30-month MIP are shown in Figures 9 and 10. This comparison shows that crew assignments are almost the same. The exception is the assignments of Crew 207 and 208 during time periods *t6* to *t10*, and all crew assignments during the final time period, *t11*.

The crew assignments during *t11* are a result of boundary effects because of information that is not available to the shorter time horizon model. For this reason, we exclude this period from analysis.

The crew assignments in Figures 9 and 10 are color-coded based on the desired crew-ship pairing shown in Table 3. This shows strong evidence of symmetry because Crew 207 and Crew 208 are in the same ship pair. If LCSS switched these assignments, the resulting schedule would be feasible and have an identical objective function value. This evidences nesting and symmetry in this case.

	t0	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11
LCS2	Crew202	Crew202	Crew202	Crew202	Crew202	Crew203	Crew203	Crew203	Crew203	Crew203	Crew203	Crew207
LCS4	Crew204	Crew204	Crew204	Crew204	Crew205	Crew205	Crew205	Crew205	Crew205	Crew205	Crew202	Crew202
LCS6	Crew203	Crew203	Crew203	Crew203	Crew203	Crew206	Crew206	Crew206	Crew206	Crew206	Crew206	Crew208
LCS8	Crew206	Crew206	Crew206	Crew206	Crew206	Crew201	Crew201	Crew201	Crew201	Crew201	Crew204	Crew204
LCS10	-	-	-	-	-	-	Crew207	Crew207	Crew207	Crew207	Crew207	Crew203
LCSa	Crew201	Crew201	Crew201	Crew201	Crew201	Crew202	Crew202	Crew202	Crew202	Crew202	Crew201	Crew201
LCSb	Crew205	Crew205	Crew205	Crew205	Crew204	Crew204	Crew204	Crew204	Crew204	Crew204	Crew205	Crew205
LCSc	-	-	-	-	-	-	Crew208	Crew208	Crew208	Crew208	Crew208	Crew206

Figure 9. 12-month, LCSS-generated schedule using MIP

	t0	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11
LCS2	Crew202	Crew202	Crew202	Crew202	Crew202	Crew203	Crew203	Crew203	Crew203	Crew203	Crew203	Crew202
LCS4	Crew204	Crew204	Crew204	Crew204	Crew205	Crew205	Crew205	Crew205	Crew205	Crew205	Crew201	Crew201
LCS6	Crew203	Crew203	Crew203	Crew203	Crew203	Crew206	Crew206	Crew206	Crew206	Crew206	Crew206	Crew207
LCS8	Crew206	Crew206	Crew206	Crew206	Crew206	Crew201	Crew201	Crew201	Crew201	Crew201	Crew204	Crew204
LCS10	-	-	-	-	-	-	Crew208	Crew208	Crew208	Crew208	Crew208	Crew203
LCSa	Crew201	Crew201	Crew201	Crew201	Crew201	Crew202	Crew202	Crew202	Crew202	Crew202	Crew202	Crew206
LCSb	Crew205	Crew205	Crew205	Crew205	Crew204	Crew204	Crew204	Crew204	Crew204	Crew204	Crew205	Crew205
LCSc	-	-	-	-	-	-	Crew207	Crew207	Crew207	Crew207	Crew207	Crew208

Figure 10. First 12 months of a 30-month, LCSS-generated schedule using MIP

2. Schedule Output

Twenty-four months is selected as a reasonable time horizon for which an operational planner can schedule with relative certainty. The resulting objective function for the 24-month MIP is $Z^{MIP,24} = 4{,}648$. Events that occur beyond 24 months typically can only be predicted from programmatic time tables such as the LCS Shipbuilding Plan (LCS Program Manager, 2013) instead of operational functions such as deployments, multi-lateral exercises and routine maintenance. However, because of the phased ship building plan employed by LCS, longer schedules are required, but cannot be solved in a reasonable amount of time using the MIP approach.

The crew assignments made by LCSS are imported into a Microsoft Excel spreadsheet (Microsoft, 2010) and generate a formatted schedule for the planner. To evaluate the performance of this schedule the operational planner first compares the crew assignments to the desired crew-ship pairing (see Table 3). This schedule (see Figure 11) has the desired attributes: crews return to their assigned pairing quickly after a move to satisfy the condition of having an experienced crew on a ship in the acceptance phase. The acceptances crews for LCS 6 and LCS 8 immediately return to their designated ship pair. However, Crew 203 is tasked to accept LCS 10 and they remain on the ship instead of returning to LCS 2 in Crew 209's place. This is due to the restriction that a crew remains in a phase for no more than six months. This swap requires Crew 209 to conduct back-to-back CONUS-On phases which extends them past the six month constraint. However, LCSS attempts to give the operational planner a starting point from which to generate a final schedule that can take into account exceptions to policy. The LCSRON Commander would be able to waive this back-to-back constraint on a case-by-case basis, a functionality that cannot be captured in our model.

	t0	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11
LCS2	Crew202	Crew202	Crew202	Crew202	Crew202	Crew201	Crew201	Crew201	Crew201	Crew201	Crew201	Crew203
	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On
LCS4	Crew204	Crew204	Crew204	Crew204	Crew205	Crew205	Crew205	Crew205	Crew205	Crew205	Crew202	Crew202
	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On
LCSa	Crew201	Crew201	Crew201	Crew201	Crew201	Crew203	Crew203	Crew203	Crew203	Crew203	Crew203	Crew206
	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off
LCS6	Crew203	Crew203	Crew203	Crew203	Crew203	Crew206	Crew206	Crew206	Crew206	Crew206	Crew206	Crew208
	Accept	Accept	Accept	Accept	Accept	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On
LCS8	Crew206	Crew206	Crew206	Crew206	Crew206	Crew202	Crew202	Crew202	Crew202	Crew202	Crew204	Crew204
	Precom	Precom	Precom	Precom	Precom	Accept	Accept	Accept	Accept	Accept	CONUS-On	CONUS-On
LCSb	Crew205	Crew205	Crew205	Crew205	Crew204	Crew204	Crew204	Crew204	Crew204	Crew204	Crew205	Crew205
	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off
LCS10	-	-	-	-	-	-	Crew207	Crew207	Crew207	Crew207	Crew207	Crew201
							Precom	Precom	Precom	Precom	Precom	Accept
LCS12	-	-	-	-	-	-	-	-	-	-	-	-
LCSc	-	-	-	-	-	-	Crew208	Crew208	Crew208	Crew208	Crew208	Crew207
							CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off
LCS14	-	-	-	-	-	-	-	-	-	-	-	-
LCS16	-	-	-	-	-	-	-	-	-	-	-	-
LCSd	-	-	-	-	-	-	-	-	-	-	-	-

	t12	t13	t14	t15	t16	t17	t18	t19	t20	t21	t22	t23
LCS2	Crew203	Crew203	Crew203	Crew203	Crew203	Crew209	Crew209	Crew209	Crew209	Crew209	Crew209	Crew209
	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	Deploy	Deploy
LCS4	Crew202	Crew202	Crew202	Crew207	Crew207	Crew207	Crew207	Crew207	Crew202	Crew202	Crew202	Crew202
	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On
LCSa	Crew206	Crew206	Crew206	Crew202	Crew202	Crew202	Crew202	Crew202	Crew201	Crew201	Crew201	Crew201
	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off
LCS6	Crew208	Crew208	Crew208	Crew206	Crew206	Crew206	Crew206	Crew206	Crew206	Crew206	Crew206	Crew204
	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	Deploy	Deploy	Deploy	Deploy	Deploy
LCS8	Crew204	Crew204	Crew204	Crew205	Crew205	Crew205	Crew205	Crew204	Crew204	Crew204	Crew204	Crew210
	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On
LCSb	Crew205	Crew205	Crew205	Crew204	Crew204	Crew204	Crew204	Crew205	Crew205	Crew205	Crew205	Crew205
	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off
LCS10	Crew201	Crew201	Crew201	Crew201	Crew201	Crew201	Crew201	Crew201	Crew208	Crew208	Crew208	Crew208
	Accept	Accept	Accept	Accept	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On	CONUS-On
LCS12	Crew209	Crew209	Crew209	Crew209	Crew209	Crew203	Crew203	Crew203	Crew203	Crew203	Crew203	Crew203
	Precom	Precom	Precom	Precom	Precom	Accept	Accept	Accept	Accept	Accept	CONUS-On	CONUS-On
LCSc	Crew207	Crew207	Crew207	Crew208	Crew208	Crew208	Crew208	Crew208	Crew207	Crew207	Crew207	Crew207
	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off
LCS14							Crew211	Crew211	Crew211	Crew211	Crew211	Crew206
							Precom	Precom	Precom	Precom	Precom	Accept
LCS16												
LCSd							Crew210	Crew210	Crew210	Crew210	Crew210	Crew211
							CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off	CONUS-Off

Figure 11. 24-month, LCSS-generated crew assignment results

B. ROLLING HORIZON RESULTS

The RH process allows the user wide flexibility in selecting the variables controlling sub-problems and time horizons. Therefore, we explore different choices for the number of sub-problems and length of the fixed, unfixed and unfixed overlap periods (see Section III.B.1) to compare the quality of the solution and the time to solve. The parameters that generate the best RH solution we have found use three sub-problems that have an unfixed overlap of four months. The first sub-problem has an unfixed period of

18 months and subsequent sub-problems have an unfixed period of 15 months, including the four-month unfixed overlap period. Each sub-problem is solved to a 5% relative optimality gap. RH generates a feasible solution in 76 minutes with an objective value of $Z^{RH,40}$=17,676.

Once again crew assignments are grouped in accordance with Table 3. A comparison of the first 30 months of the 40-month RH solution (see Figure 13) against the 30-month MIP (see Figure 12) reveals differences in group assignments. The equivalent objective function value for the RH solution for the first 30 months is $Z^{RH,30}$=7,987. This is higher than the 30-month MIP value in Table 4 ($Z^{MIP,30}$=5,004), which is expected because RH is a heuristic approach.

The RH schedule changes the desired crew-ship pairings from Crews 204-206 and Crews 207-209. Figure 13 shows that at *t28*, Crews 204-206 are the only crews assigned to LCS 2 and LCS 4 even though Table 3 puts them on LCS 6 and LCS 8. Similarly, Crews 207-209 are assigned to LCS 6 and LCS 8 beginning at *t15* until the end of the 30-month schedule, when they should be assigned to LCS 10 and LCS 12. The myopic approach of each sub-problem of RH makes it hard for LCSS to return to the steady-state crew assignment desires of LCSRON. This can be mitigated by choosing a larger overlap at the expense of driving solve times higher.

C. FIX AND RELAX RESULTS

F&R is run with three sub-problems where the first sub-problem has an Integer subset of 18 months and a Continuous subset of 22 months. The second sub-problem has a Fixed subset of 14 months, an Integer subset of 18 months, an unfixed overlap of four months and a Continuous subset of eight months. The third sub-problem has a Fixed subset of 29 months, an Integer subset of 11 months and an unfixed overlap of four months. Each sub-problem is solved to a 5% relative optimality gap. F&R generates a feasible solution in 7 hours with an objective value of $Z^{F\&R,40}$=14,382 (improving the RH solution, $Z^{RH,40}$=17,676)

Once again crew assignments are grouped in accordance with Table 3. A comparison of the first 30 months of the 40-month F&R solution (see Figure 14) against the 30-month MIP (see Figure 12) reveals differences in group assignments. The equivalent objective function value for the F&R solution for the first 30 months is $Z^{F\&R,30}$=9,856. This is again higher than the 30-month MIP value in Table 4, which is expected because F&R is a heuristic approach and because the F&R solution is also taking into accounts events to occur up to month 40. It is slightly counter-intuitive (albeit possible) that the F&R solution up to month 30 is worse than the RH solution up to that period. This can be explained, again, because each F&R sub-problem is planning for up to 40 months, whereas the RH sub-problems are optimized to shorter horizons of 18, 29 and 40 months. The F&R schedule assigns both Crew 204 and 205 to the CONUS-Off phase between *t26* and *t29*. This results in a significant penalty, but it is used to better satisfy future requirements: Crew 204 returning to its designated pairing at *t31* to finish its training cycle before deploying on LCS 8 at *t35*.

D. SOLUTION COMPARISONS

In this section, we summarize the comparisons for solutions returned from the three approaches. Comparison are based on penalty value and solve times (see Table 5), and on how an operational planner can use the results. We define two likely scenarios faced by a planner: Scenario 1 is the routine changes to a schedule that occur because of unscheduled maintenance, meteorological delays or shipyard delays; Scenario 2 is the robust analysis of alternatives required when considering the impact of long-term commitments like exercise participation or extended deployments. The acceptable time to generate a solution is set at the overnight time between two working days, or 12 hours.

Table 5. Comparison of LCSS penalty values and solve times for MIP, RH and F&R

Method	**30-month MIP**	**40-month RH**			**40-month F&R**		
Period compared	30 months	30 mo.	36 mo.	40 mo.	30 mo.	36 mo.	40 mo.
Objective Value	5,004	7,987	16,263	17,676	9,856	12,147	14,382
Solve Time	6 hrs 35 min	1 hr 16 min			7 hrs		

RH returns a three-year schedule in the shortest time, 76 minutes. In the face of Scenario 1 the planner can quickly generate a starting schedule and manually adjust it before recommending a schedule. However, when faced with Scenario 2, RH does not keep crews in their desired pairings well and numerous manual changes are required in each schedule before the alternatives can be compared.

F&R takes seven hours to return a feasible solution, which makes this technique appropriate for Scenario 1. However, an analysis of alternatives in Scenario 2 would require running multiple instances of LCSS simultaneously or accept a multi-day delay in planning.

The MIP returns a feasible solution in less than 12 hours if 30 or fewer months are considered. If schedule changes in Scenario 1 are short-term, it may be acceptable to solve the full MIP with a short time horizon to generate solutions that require minimal manual changes. However, long-range schedule changes or the long-term impact of multiple schedules cannot be solved by LCSS using the full MIP within a reasonable time for the operational planner.

Comparing all three approaches on a short time horizon (less than 18 months) shows that the assignment decisions are symmetrical. F&R assignments match more closely with the MIP than do the RH results for the short-term assignments. However, truncating the 40-month F&R solution to 30 months for comparison purposes led to overall poorer performance, compared to RH. Comparing longer time horizon schedules reveals that F&R begins to outperform RH for longer-term schedules (see Table 5). The operational planner must consider the task at hand and choose the appropriate approach. If the schedule changes are short-term only, then a satisfactory MIP solution can be generated. Further, if the short-term schedule changes do not occur frequently, the longer computational time required by the MIP approach may be acceptable to the planner. However, if long-term ship schedules play a major role in the decision making process, then F&R ability to consider out-year schedules allows for better long-term performance.

	t0	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11	t12	t13	t14	t15	t16	t17	t18	t19	t20	t21	t22	t23	t24	t25	t26	t27	t28	t29
LCS2	Crew202	Crew202	Crew202	Crew202	Crew202	Crew203	Crew203	Crew203	Crew203	Crew203	Crew203	Crew202	Crew202	Crew202	Crew202	Crew202	Crew202	Crew209	Crew209	Crew209	Crew209	Crew209	Crew209	Crew209	Crew209	Crew209	Crew209	Crew209	Crew203	Crew203
LCS4	Crew204	Crew204	Crew204	Crew204	Crew205	Crew205	Crew205	Crew205	Crew205	Crew205	Crew201	Crew201	Crew201	Crew201	Crew201	Crew208	Crew208	Crew208	Crew208	Crew208	Crew201	Crew201	Crew201	Crew201	Crew203	Crew203	Crew203	Crew203	Crew201	Crew201
LCS6	Crew203	Crew203	Crew203	Crew203	Crew203	Crew206	Crew206	Crew206	Crew206	Crew206	Crew206	Crew207	Crew207	Crew207	Crew207	Crew206	Crew206	Crew206	Crew206	Crew206	Crew206	Crew206	Crew206	Crew210	Crew210	Crew210	Crew210	Crew210	Crew210	Crew208
LCS8	Crew206	Crew206	Crew206	Crew206	Crew206	Crew201	Crew201	Crew201	Crew201	Crew201	Crew204	Crew204	Crew204	Crew204	Crew205	Crew205	Crew205	Crew205	Crew210	Crew210	Crew210	Crew210	Crew210	Crew205	Crew205	Crew205	Crew205	Crew204	Crew204	Crew204
LCS10	-	-	-	-	-	-	Crew208	Crew208	Crew208	Crew208	Crew208	Crew203	Crew203	Crew203	Crew203	Crew203	Crew203	Crew203	Crew203	Crew203	Crew207	Crew207	Crew207	Crew207	Crew208	Crew208	Crew208	Crew208	Crew208	Crew212
LCS12	-	-	-	-	-	-	-	-	-	-	-	-	Crew209	Crew209	Crew209	Crew209	Crew209	Crew202	Crew202	Crew202	Crew202	Crew202	Crew202	Crew202	Crew202	Crew202	Crew202	Crew202	Crew207	Crew207
LCS14	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Crew204	Crew204	Crew204	Crew204	Crew204	Crew206	Crew206	Crew206	Crew206	Crew206	Crew206	Crew206
LCS16	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Crew211	Crew211	Crew211	Crew211	Crew211	Crew210
LCSa	Crew201	Crew201	Crew201	Crew201	Crew201	Crew202	Crew202	Crew202	Crew202	Crew202	Crew202	Crew206	Crew206	Crew206	Crew206	Crew201	Crew201	Crew201	Crew201	Crew201	Crew203	Crew203	Crew203	Crew203	Crew201	Crew201	Crew201	Crew201	Crew202	Crew202
LCSb	Crew205	Crew205	Crew205	Crew205	Crew204	Crew204	Crew204	Crew204	Crew204	Crew204	Crew205	Crew205	Crew205	Crew205	Crew204	Crew204	Crew204	Crew204	Crew205	Crew205	Crew205	Crew205	Crew205	Crew204	Crew204	Crew204	Crew204	Crew205	Crew205	Crew205
LCSc	-	-	-	-	-	-	Crew207	Crew207	Crew207	Crew207	Crew207	Crew208	Crew208	Crew208	Crew208	Crew207	Crew207	Crew207	Crew207	Crew207	Crew208	Crew208	Crew208	Crew208	Crew207	Crew207	Crew207	Crew207	Crew209	Crew209
LCSd	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Crew211	Crew211	Crew211	Crew211	Crew211	Crew211	Crew212	Crew212	Crew212	Crew212	Crew212	Crew211
LCSe	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

Figure 12. 30-month LCSS generated schedule using Mixed Integer Program Formulation

	t0	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11	t12	t13	t14	t15	t16	t17	t18	t19	t20	t21	t22	t23	t24	t25	t26	t27	t28	t29	t30	t31	t32	t33	t34	t35	t36
LCS2	Crew202	Crew202	Crew202	Crew202	Crew202	Crew203	Crew203	Crew203	Crew203	Crew203	Crew203	Crew202	Crew202	Crew202	Crew202	Crew202	Crew202	Crew204	Crew204	Crew204	Crew204	Crew204	Crew202	Crew202	Crew202	Crew202	Crew205	Crew205	Crew205	Crew205	Crew204	Crew204	Crew204	Crew204	Crew205	Crew205	Crew205
LCS4	Crew204	Crew204	Crew204	Crew204	Crew205	Crew205	Crew205	Crew205	Crew204	Crew204	Crew204	Crew204	Crew205	Crew205	Crew205	Crew205	Crew205	Crew209	Crew209	Crew209	Crew209	Crew209	Crew209	Crew210	Crew210	Crew210	Crew210	Crew210	Crew206	Crew206	Crew206	Crew206	Crew206	Crew206	Crew206	Crew206	Crew206
LCS6	Crew203	Crew203	Crew203	Crew203	Crew203	Crew206	Crew206	Crew206	Crew206	Crew206	Crew206	Crew208	Crew208	Crew208	Crew208	Crew206	Crew206	Crew206	Crew206	Crew206	Crew206	Crew206	Crew206	Crew208	Crew208	Crew208	Crew208	Crew207	Crew207	Crew207	Crew207	Crew208	Crew208	Crew208	Crew208	Crew214	Crew214
LCS8	Crew206	Crew206	Crew206	Crew206	Crew206	Crew201	Crew201	Crew201	Crew201	Crew201	Crew201	Crew201	Crew201	Crew201	Crew201	Crew207	Crew207	Crew207	Crew207	Crew208	Crew208	Crew208	Crew208	Crew207	Crew207	Crew207	Crew207	Crew209	Crew209	Crew209	Crew209	Crew209	Crew209	Crew209	Crew209	Crew202	Crew202
LCS10	-	-	-	-	-	-	Crew207	Crew207	Crew207	Crew207	Crew207	Crew203	Crew203	Crew203	Crew203	Crew203	Crew203	Crew203	Crew203	Crew203	Crew201	Crew201	Crew201	Crew201	Crew212	Crew212	Crew212	Crew212	Crew212	Crew212	Crew212	Crew212	Crew212	Crew212	Crew212	Crew212	Crew210
LCS12	-	-	-	-	-	-	-	-	-	-	-	-	Crew209	Crew209	Crew209	Crew209	Crew209	Crew202	Crew202	Crew202	Crew202	Crew202	Crew205	Crew205	Crew205	Crew205	Crew204	Crew204	Crew204	Crew204	Crew202	Crew202	Crew202	Crew202	Crew202	Crew207	Crew207
LCS14	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Crew211	Crew211	Crew211	Crew211	Crew211	Crew206	Crew206	Crew206	Crew206	Crew206	Crew211	Crew211	Crew211	Crew211	Crew210	Crew210	Crew210	Crew210	Crew215
LCS16	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Crew203	Crew203	Crew203	Crew203	Crew203	Crew201	Crew201	Crew201	Crew201	Crew201	Crew203	Crew203	Crew203
LCS18	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Crew213	Crew213	Crew213	Crew213	Crew213	Crew208	Crew208
LCS20	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Crew211
LCSa	Crew201	Crew201	Crew201	Crew201	Crew201	Crew202	Crew202	Crew202	Crew202	Crew202	Crew202	Crew206	Crew206	Crew206	Crew206	Crew201	Crew201	Crew201	Crew201	Crew201	Crew203	Crew203	Crew203	Crew203	Crew201	Crew201	Crew201	Crew201	Crew201	Crew203	Crew203	Crew203	Crew203	Crew203	Crew201	Crew201	Crew201
LCSb	Crew205	Crew205	Crew205	Crew205	Crew204	Crew204	Crew204	Crew204	Crew205	Crew205	Crew205	Crew205	Crew204	Crew204	Crew204	Crew204	Crew204	Crew205	Crew205	Crew205	Crew205	Crew205	Crew204	Crew204	Crew204	Crew204	Crew202	Crew202	Crew202	Crew202	Crew205	Crew205	Crew205	Crew205	Crew204	Crew204	Crew204
LCSc	-	-	-	-	-	-	Crew208	Crew208	Crew208	Crew208	Crew208	Crew207	Crew207	Crew207	Crew207	Crew208	Crew208	Crew208	Crew208	Crew207	Crew207	Crew207	Crew207	Crew209	Crew209	Crew209	Crew209	Crew208	Crew208	Crew208	Crew208	Crew207	Crew207	Crew207	Crew207	Crew209	Crew209
LCSd	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Crew210	Crew210	Crew210	Crew210	Crew210	Crew211	Crew211	Crew211	Crew211	Crew211	Crew210	Crew210	Crew210	Crew210	Crew211	Crew211	Crew211	Crew211	Crew212
LCSe	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Crew214	Crew214	Crew214	Crew214	Crew214	Crew213	Crew213

Figure 13. First 36 months of a 40-month LCSS generated schedule using Rolling Horizon

	t0	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11	t12	t13	t14	t15	t16	t17	t18	t19	t20	t21	t22	t23	t24	t25	t26	t27	t28	t29	t30	t31	t32	t33	t34	t35	t36
LCS2	Crew202	Crew202	Crew202	Crew202	Crew202	Crew201	Crew201	Crew201	Crew201	Crew201	Crew201	Crew203	Crew203	Crew203	Crew203	Crew203	Crew203	Crew209	Crew209	Crew209	Crew209	Crew209	Crew203	Crew203	Crew203	Crew203	Crew203	Crew203	Crew206	Crew206	Crew206	Crew206	Crew206	Crew206	Crew203	Crew203	Crew203
LCS4	Crew204	Crew204	Crew204	Crew204	Crew205	Crew205	Crew205	Crew205	Crew205	Crew205	Crew202	Crew202	Crew202	Crew202	Crew202	Crew208	Crew208	Crew208	Crew208	Crew208	Crew202	Crew202	Crew202	Crew202	Crew201	Crew201	Crew201	Crew201	Crew201	Crew212	Crew212	Crew212	Crew212	Crew202	Crew202	Crew202	Crew202
LCS6	Crew203	Crew203	Crew203	Crew203	Crew203	Crew206	Crew206	Crew206	Crew206	Crew206	Crew206	Crew207	Crew207	Crew207	Crew207	Crew206	Crew206	Crew206	Crew206	Crew206	Crew206	Crew206	Crew206	Crew204	Crew204	Crew204	Crew204	Crew205	Crew205	Crew205	Crew205	Crew204	Crew204	Crew204	Crew204	Crew214	Crew214
LCS8	Crew206	Crew206	Crew206	Crew206	Crew206	Crew202	Crew202	Crew202	Crew202	Crew202	Crew204	Crew204	Crew204	Crew204	Crew205	Crew205	Crew205	Crew205	Crew204	Crew204	Crew204	Crew204	Crew204	Crew210	Crew210	Crew210	Crew210	Crew211	Crew211	Crew211	Crew211	Crew211	Crew211	Crew211	Crew211	Crew211	Crew211
LCS10	-	-	-	-	-	-	Crew208	Crew208	Crew208	Crew208	Crew208	Crew201	Crew201	Crew201	Crew201	Crew201	Crew201	Crew201	Crew201	Crew201	Crew207	Crew207	Crew207	Crew207	Crew207	Crew208	Crew208	Crew208	Crew208	Crew208	Crew208	Crew208	Crew208	Crew208	Crew208	Crew207	Crew207
LCS12	-	-	-	-	-	-	-	-	-	-	-	-	Crew209	Crew209	Crew209	Crew209	Crew209	Crew203	Crew203	Crew203	Crew203	Crew203	Crew205	Crew205	Crew205	Crew205	Crew205	Crew209	Crew209	Crew209	Crew209	Crew207	Crew207	Crew207	Crew207	Crew209	Crew209
LCS14	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Crew211	Crew211	Crew211	Crew211	Crew211	Crew206	Crew206	Crew206	Crew206	Crew206	Crew202	Crew202	Crew202	Crew202	Crew202	Crew210	Crew210	Crew210	Crew210
LCS16	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Crew212	Crew212	Crew212	Crew212	Crew212	Crew201	Crew201	Crew201	Crew201	Crew201	Crew213	Crew213	Crew213
LCS18	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Crew214	Crew214	Crew214	Crew214	Crew214	Crew205	Crew205
LCS20	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Crew215
LCS22																																					
LCS24																																					
LCSa	Crew201	Crew201	Crew201	Crew201	Crew201	Crew203	Crew203	Crew203	Crew203	Crew203	Crew203	Crew206	Crew206	Crew206	Crew206	Crew202	Crew202	Crew202	Crew202	Crew202	Crew201	Crew201	Crew201	Crew201	Crew202	Crew202	Crew202	Crew202	Crew203	Crew203	Crew203	Crew203	Crew203	Crew203	Crew201	Crew201	Crew201
LCSb	Crew205	Crew205	Crew205	Crew205	Crew204	Crew204	Crew204	Crew204	Crew204	Crew204	Crew205	Crew205	Crew205	Crew205	Crew204	Crew204	Crew204	Crew204	Crew205	Crew205	Crew205	Crew205	Crew209	Crew209	Crew209	Crew209	Crew209	Crew204	Crew204	Crew204	Crew204	Crew205	Crew205	Crew205	Crew205	Crew204	Crew204
LCSc	-	-	-	-	-	-	Crew207	Crew207	Crew207	Crew207	Crew207	Crew208	Crew208	Crew208	Crew208	Crew207	Crew207	Crew207	Crew207	Crew207	Crew208	Crew208	Crew208	Crew208	Crew208	Crew207	Crew207	Crew207	Crew207	Crew207	Crew207	Crew209	Crew209	Crew209	Crew209	Crew208	Crew208
LCSd	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Crew210	Crew210	Crew210	Crew210	Crew210	Crew211	Crew211	Crew211	Crew211	Crew210	Crew210	Crew210	Crew210	Crew210	Crew210	Crew212	Crew212	Crew212	Crew212
LCSe	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Crew213	Crew213	Crew213	Crew213	Crew206	Crew206	Crew206

Figure 14. First 36 months of a 40-month LCSS generated schedule using Fix and Relax

V. CONCLUSIONS AND RECOMMENDATIONS

This thesis has developed LCSS, an optimization tool that reduces the initial workload of the LCSRON staff scheduler. LCSS seeks to minimize the penalty associated with assigning a crew to a ship outside of its desired ship pairing and/or extending a crew beyond four months in a phase.

The size of the problem makes solving a five-year schedule using the full MIP computationally intractable. Our MIP can produce up to a 30-month schedule, and we use RH and F&R heuristics to produce 36-month solutions. RH generates solutions in the shortest amount of time, but those schedules do not keep crews in their desired pairings. F&R produces superior long-term schedules when compared to a similar-length RH schedule. However, the short-term crew assignments do not appear as ideal. LCSS MIP solve times increase exponentially making longer range schedules require too much computational time to be useful for the scheduler. However, if the scheduler desires an updated short-term schedule, MIP provides quick, and guaranteed optimal, results that require minimal manual changes.

The assignment cost in LCSS is fixed across all time periods for a given crew-ship pair. In practice, there are situations when crews have differing penalty costs over time based on individual operational tempo, leadership changes or political sensitivities. Additionally, the uncertainty of future schedules could be captured by discounting the penalty value for future months (e.g., after the second year). This would weight short-term assignments more than long-term and allow more future disruption to crew pairings before LCSS would change short-term assignments. Future versions of LCSS can incorporate these to yield a better initial solution for the operational planner.

LCSS uses a predetermined ship schedule to assign crews. It is difficult to manually determine if small changes to a ship's schedule will have a dramatic improvement in the overall crew schedule. However, if a better crew-ship schedule exists, a commander has the prerogative to make such a change. Therefore, it would be beneficial to allow LCSS to recommend changes to the ship schedule within pre-defined parameters.

F&R is not limited to partitioning sub-problems on the basis of time. Restructuring the F&R implementation partitioned on ships may improve performance by decreasing solve time. In that case, the usefulness to an operational planner will be increased.

We have not implemented persistence in LCSS, but considering the potential changes in ship production schedules, we deem it would be a useful extension of this work for the operational planner.

In the end, LCSS is a tool designed to allow an operational planner to focus on the intangible details of crew scheduling. The fusion of a mathematically optimal schedule based on tangible considerations with the complexity of real-world scheduling will allow the LCSS to provide a robust, flexible response to Navy requirements around the globe.

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